MODULE 9 GREATEST COMMON DIVISOR

BUILDING UNDERSTANDING

A ***divisor*** is a number or quantity by which a dividend is divided to produce a quotient. In the situation where 50 is to be divided by 10, 50 is the *dividend*, 10 is the *divisor* and 5 is the *quotient*.

A ***common divisor*** is a divisor of more than one number. For example, 4 is a divisor of 48 (48 ÷ 4 = 12) and 4 is also a divisor of 36 (36 ÷ 4 = 9). Therefore, 4 is one common divisor for 48 and 36. However, 48 and 36 have other common divisors including 2, 6, and 12.

The largest common divisor of two numbers is called their ***greatest common divisor*** (GCD). The greatest common divisor of 48 and 36 is 12. In other words, 12 is the largest number than can be evenly divided into both 48 and 36.

METHODOLOGY

The following is a method for finding the greatest common divisor of two numbers. It involves finding the prime factors for each number and placing these factors into a table where it becomes easy to determine which numbers are to be used to calculate the greatest common divisor.

To Find the Greatest Common Divisor (GCD):

1. Determine the prime factors for each number. Refer to the previous module "Factoring Natural Numbers" for more information.
2. Create the following table.

	prime number #1	prime number #2	prime number #3	***continue with as many prime numbers as there are for the numbers***
first number	list all occurrences	list all occurrences	list all occurrences	
second number	list all occurrences	list all occurrences	list all occurrences	
common occurrences	list all common occurrences	list all common occurrences	list all common occurrences	

   a) Place the numbers for which we are finding the GCD in the first column.
   b) List all the prime numbers that are factors for the numbers in the first column. Place each prime number at the top of a column.
   c) Under each prime number, list all occurrences of that prime number when factoring each number in the first column.
   d) In the row titled "common occurrences," list all the common occurrences of prime numbers.This represents the greatest number of occurrences of that particular prime number.

3. Calculate the greatest common divisor by multiplying all the prime numbers that are listed in the "common occurrences" row. The product of these numbers is the GCD.

Models

Find the greatest common divisor for the following pairs of numbers.

GCD for 10 and 12:

Prime factors for each number are: $10 = 2 \times 5$ and $12 = 2 \times 2 \times 3$

	prime number: 2	prime number: 3	prime number: 5
first number: 10	2		5
second number: 12	2, 2	3	
common occurrences	2		

GCD = 2

GCD for 30 and 45:

Prime factors for each number are: $30 = 2 \times 3 \times 5$ and $45 = 3 \times 3 \times 5$

	prime number: 2	prime number: 3	prime number: 5
first number: 30	2	3	5
second number: 45		3, 3	5
common occurrences		3	5

GCD = 3 × 5 = 15

GCD for 27 and 45:

Prime factors for each number are: $27 = 3 \times 3 \times 3$ and $45 = 3 \times 3 \times 5$

	prime number: 3	prime number: 5
first number: 27	3, 3, 3	
second number: 45	3, 3	5
common occurrences	3, 3	

GCD = 3 × 3 = 9

GCD for 126 and 315:

Prime factors for each number are: $126 = 2 \times 3 \times 3 \times 7$ and $315 = 3 \times 3 \times 5 \times 7$

	prime number: 2	prime number: 3	prime number: 5	prime number: 7
first number: 126	2	3, 3		7
second number: 315		3, 3	5	7
common occurrences		3, 3		7

GCD = 3 × 3 × 7 = 9 × 7 = 63

Critical Thinking Questions

1. What does it mean for a number to be the greatest common divisor of two numbers?

2. What is the difference between a common divisor and a greatest common divisor?

3. Can the GCD of a set of numbers be larger than any of the numbers? Why?

4. Must all the factors for every number in a set of numbers be evenly divisible by the GCD? Why?

5. How can you validate that the GCD you've found is correct?