Section 2.2 Solving Basic Equations

Why? 2.2

You can solve some equations that arise in the real world by isolating a variable. You can use this method to solve the equation

$$400+\left(1\frac{1}{2}\right)(10)x=460$$

to determine the number of hours of overtime you need to work to earn \$460 per week, at an hourly rate of \$10, with overtime being time and a half.

What Do You Already Know? 2.2

Give two examples of pairs of equivalent equations.

Use the Addition Property of Equality to create an equation that is equivalent to $6+3x=5$.

Use the Multiplication Property of Equality to create an equation that is equivalent to $-3(x-5)=6$.

Use the Distributive Property and the Substitution Principle to create an equation that is equivalent to $5x+2(3-x)=10$.

Goals 2.2

When you complete this activity, you should be able to:

1. Solve a linear equation using the Isolate the Variable Methodology
2. Solve a literal equation
3. Use the Solving Linear Equations Methodology to find the solution to a linear equation involving fractions and parentheses

Building Mathematical Language 2.2

New Terms: isolate a variable literal equation

Key Concepts 2.2

Some equations have *no solutions* and some equations have an *infinite number of solutions*. These occur when the variable can be totally eliminated from the equation. The resulting equation will state that two numbers are equal. If the two numbers are different, then the equation is never true and there are no solutions to the original equation. If the two numbers are the same, the equation is always true and there are an infinite number of solutions.

A ***literal equation*** is an equation with more than one variable. Engineers, scientists, and government policy makers use these equations because they provide information about relationships between real world quantities such as population and the square feet of retail space required.

Limitation: When dealing with literal equations, you must take care to understand the constraints on the variables in the equations. Assuming that literal equations are true for all values of their variables can cause safety issues in energy plants, on planes, and in factories.

Methodology 2.2

Isolating a Variable

Isolating a variable in an equation with no fractions or parentheses, and where the variable appears only to the first power, is useful in solving an equation.

Limitation/Caution: If a term has the specified variable in the denominator, then problems can occur (we will see this later when we work with rational equations).

Example 1	Example 2	Your Turn
Isolate x: $6x - 5 = 7 + 2x$	Isolate x: $6x + 5 - x = 3x + 9$	Isolate a: $2a + 4a = 7a + 2 - 3a$

Steps	Discussion
1 Rewrite the equation	**Using the Equality Properties, rewrite the equation so that all the terms of your chosen variable are on one side of the equation and all the other terms on the other side of the equation.**

Ex 1	Ex 2	Your Turn
$6x - 2x = 7 + 5$	$6x - x - 3x = 9 - 5$	

2 Simplify the expressions	**Simplify the expressions on both sides of the equation.**

Ex 1	Ex 2	Your Turn
$4x = 12$	$5x - 3x = 4$ $2x = 4$	

3 Rewrite the equation	**Use the Distributive Property to rewrite the equation so that the chosen variable is a factor.**

Ex 1	Ex 2	Your Turn
Done	Done	

4 Divide by the coefficient	**Divide both sides of the equation by the coefficient of the chosen variable.**

Ex 1	Ex 2	Your Turn
$4x = 12$ $x = 3$	$2x = 4$ $x = 2$	

Steps	Discussion
5 Validate	**Substitute the value of the isolated variable in the equation to verify that the chosen variable has been isolated correctly.**

Ex 1

$$6x - 5 = 7 + 2x$$
$$6(3) - 5 \stackrel{?}{=} 7 + 2(3)$$
$$18 - 5 \stackrel{?}{=} 7 + 6$$
$$13 = 13 \checkmark$$

Ex 2

$$6x + 5 - x = 3x + 9$$
$$6(2) + 5 - 2 \stackrel{?}{=} 3(2) + 9$$
$$12 + 5 - 2 \stackrel{?}{=} 6 + 9$$
$$17 - 2 \stackrel{?}{=} 15$$
$$15 = 15 \checkmark$$

Your Turn

Model 1: Isolating a Variable in a Literal Equation 2.2

Isolate y^2: $3y^2 - 4ax = 7ax - 5 + ay^2$

Step 1 **Rewrite the equation** $3y^2 - ay^2 = 7ax - 5 + 4ax$

Step 2 **Simplify** $3y^2 - ay^2 = 11ax - 5$

Step 3 **Rewrite with chosen variable as a factor** $y^2(3 - a) = 11ax - 5$

Step 4 **Divide by the coefficient**

$$\frac{y^2(3-a)}{(3-a)} = \frac{11ax - 5}{(3-a)}$$

$$y^2 = \frac{11ax - 5}{3 - a}$$

Step 5 Validate

$$3y^2 - 4ax = 7ax - 5 + ay^2$$

$$3\left(\frac{11ax-5}{3-a}\right) - 4ax = 7ax - 5 + a\left(\frac{11ax-5}{3-a}\right)$$

$$\frac{33ax-15}{3-a} - 4ax\left(\frac{3-a}{3-a}\right) = 7ax\left(\frac{3-a}{3-a}\right) - 5\left(\frac{3-a}{3-a}\right) + \frac{11a^2x-5a}{3-a}$$

$$\frac{33ax-15-12ax+4a^2x}{3-a} = \frac{21ax-7a^2x-15+5a+11a^2x-5a}{3-a}$$

$$\frac{4a^2x+21ax-15}{3-a} = \frac{4a^2x+21ax-15}{3-a} \quad \checkmark$$

Methodology 2.2

Clearing Fractions

Equations without denominators (fractions) are easier to work with.

Limitation: Some denominators with variables can cause problems.

	Example 1	Your Turn
Clear the fractions:	$\frac{x}{3} + \frac{2x}{5} = 4$	$\frac{3a}{4} + \frac{a^2}{5} = 3$

Steps	Discussion
1 Determine LCD	**Determine the LCD of all the denominators of the equation.**

Ex 1

$$\frac{x}{3} + \frac{2x}{5} = 4$$

The LCD is 3 • 5

Your Turn

2 Multiply and simplify	**Multiply both sides of the equation by the LCD of the denominators in the equation and simplify.**

Ex 1

$$15\left(\frac{x}{3} + \frac{2x}{5}\right) = 15(4)$$

$$15\left(\frac{x}{3}\right) + 15\left(\frac{2x}{5}\right) = 60$$

$$\frac{15x}{3} + \frac{30x}{5} = 60$$

$$5x + 6x = 60$$

Your Turn

Methodology 2.2

Solving a Linear Equation

A linear equation in one variable defines possible values for that variable, and can be used to find these values. Linear equations that state a relationship between two or more variables can be solved to determine the relationship one variable has to the other variable(s).

Limitation/Caution: Although some relationships defined by data appear linear for a range of values of the variable(s), assuming that a relationship is linear can lead to errors.

Solve the linear equation for the designated variable:

Example 1	Example 2	Your Turn
for x: $2(x-3)=4x-2$	for a: $2a-\frac{5}{3}a-2=3(2-a)-\frac{1}{2}$	for t: $6-3(t-5)=2t+11$

Steps	Discussion
1 Choose the variable to solve for	**Note the variables and constants and select which variable to solve for.**

Ex 1	The variable is x.	**Ex 2**	The variable is a.	**Your Turn**	

2 Clear parentheses	**All variable occurrences should be outside parentheses; carry out any operations required to clear parentheses.**

Ex 1	$2x-6=4x-2$	**Ex 2**	$2a-\frac{5}{3}a-2=6-3a-\frac{1}{2}$	**Your Turn**	

3 Clear fractions	**Multiply both sides of the equation by the LCD of the denominators and simplify (*see the Methodology for Clearing Fractions earlier n this section*).**

Ex 1	No fractions	**Ex 2**	LCD is 6 $6\left(2a-\frac{5}{3}a-2\right)=6\left(6-3a-\frac{1}{2}\right)$ $12a-10a-12=36-18a-3$

Your Turn	

Steps	Discussion
4 Simplify	**Simplify the expressions on both sides of the equation, combining like terms whenever possible.**

Ex 1 Simplified

Ex 2 $12a - 10a - 12 = 33 - 18a$

Your Turn

5 Isolate the variable	**Rewrite so that all instances of the chosen variable are on one side, simplify, and rewrite the equation so that the chosen variable is a factor. Finally, divide by any coefficient (*see the Methodology for Isolating a Variable earlier in this section*).**

Ex 1

$$2x - 6 + 6 = 4x - 2 + 6$$
$$2x = 4x + 4$$
$$2x - 4x = 4x - 4x + 4$$
$$-2x = 4$$
$$x = -2$$

Ex 2

$$(12a) - (10a) - 12 = 33 - (18a)$$
$$12a - 10a + 18a = 33 + 12$$
$$20a = 45$$
$$a = \frac{45}{20}$$
$$a = \frac{9}{4}$$

Your Turn

6 Validate	**Substitute the solution into the original equation to check that both sides of the equation are the same.**

Ex 1

$$2(x - 3) = 4x - 2$$
$$2(-2 - 3) \stackrel{?}{=} 4(-2) - 2$$
$$2(-5) \stackrel{?}{=} -8 - 2$$
$$-10 = -10 \quad \checkmark$$

Ex 2

$$2a - \frac{5}{3}a - 2 = 3(2 - a) - \frac{1}{2}$$
$$2\left(\frac{9}{4}\right) - \frac{5}{3}\left(\frac{9}{4}\right) - 2 \stackrel{?}{=} 3\left(2 - \left(\frac{9}{4}\right)\right) - \frac{1}{2}$$
$$\frac{9}{2} - \frac{15}{4} - 2 \stackrel{?}{=} 6 - \frac{27}{4} - \frac{1}{2}$$
$$\frac{18}{4} - \frac{15}{4} - \frac{8}{4} \stackrel{?}{=} \frac{24}{4} - \frac{27}{4} - \frac{2}{4}$$
$$-\frac{5}{4} = -\frac{5}{4} \quad \checkmark$$

(Or validate with a calculator.)

Steps	Discussion
Your Turn	

Model 2: Solving a Literal Equation for a Given Variable 2.2

Solve the following literal equation for r:

$$\frac{7}{3} - 2rw = \pi r$$

Step 1 Choose the variable to solve for

The variables are r and w. The symbol π is a constant. We are to solve for r.

Step 2 Clear parentheses

There are no parentheses.

Step 3 Clear fractions

$$\frac{7}{3} - 2rw = \pi r$$

LCD is 3

$$7 - 6rw = 3\pi r$$

Step 4 Simplify

There is nothing to simplify.

Step 5 Isolate the variable

$$7 - (6rw) = (3\pi r)$$

$$7 = 3\pi r + 6rw$$

$$7 = (3\pi + 6w)r$$

$$\frac{7}{(3\pi + 6w)} = r$$

$$\frac{7}{3(\pi + 2w)} = r \quad \text{or} \quad r = \frac{7}{3(\pi + 2w)}$$

Step 6 Validate

$$\frac{7}{3} - 2rw = \pi r$$

$$\frac{7}{3} - 2\left(\frac{7}{3(\pi + 2w)}\right) \bullet w = \pi\left(\frac{7}{3(\pi + 2w)}\right)$$

$$\frac{7}{3} - \frac{14w}{3(\pi + 2w)} = \frac{7\pi}{3(\pi + 2w)}$$

$$\frac{7(\pi + 2w) - 14w}{3(\pi + 2w)} = \frac{7\pi}{3(\pi + 2w)}$$

$$\frac{7\pi + 14w - 14w}{3(\pi + 2w)} = \frac{7\pi}{3(\pi + 2w)}$$

$$\frac{7\pi}{3(\pi + 2w)} = \frac{7\pi}{3(\pi + 2w)} \quad \checkmark$$

Addressing Common Errors 2.2

The following table shows some of the most common errors that learners tend to make with the kinds of problems covered in this section.

ERROR: **Incorrect use of the Distributive Property to remove parentheses**

Incorrect Process

$$3x - (2x + 4) = -2$$
$$3x - 2x + 4 = -2$$
$$x + 4 = -2$$
$$x = -6$$

Resolution

Remove the parentheses, correctly using the Distributive Property.

Correct Process

$$3x - (2x + 4) = -2$$
$$3x - 2x - 4 = -2$$
$$x - 4 = -2$$
$$x = 2$$

Validation

$$3x - (2x + 4) = -2$$
$$3(2) - (2(2) + 4) \stackrel{?}{=} -2$$
$$6 - (4 + 4) \stackrel{?}{=} -2$$
$$6 - 8 \stackrel{?}{=} -2$$
$$-2 = -2 \quad \checkmark$$

ERROR: **Incorrect use of the Addition Property of Equality**

Incorrect Process

$$3x + 7 = 11$$
$$3x = 18$$
$$x = 6$$

Resolution

Correct use of the Addition Property of Equality by subtracting 7 from both sides.

ERROR: **Incorrect use of the Addition Property of Equality (con't)**

Correct Process

$$3x+7=11$$
$$3x=4$$
$$x=\frac{4}{3}$$

Validation

$$3x+7=11$$
$$3\bullet\left(\frac{4}{3}\right)+7\overset{?}{=}11$$
$$4+7\overset{?}{=}11$$
$$11=11\ \checkmark$$

ERROR: **Incorrect use of the Multiplication Property of Equality**

Incorrect Process

$$3x=5$$
$$x=15$$

Resolution

Use the Multiplication Property of Equality correctly by multiplying both sides of the equation by $\frac{1}{3}$

Correct Process

$$3x=5$$
$$\frac{1}{3}\bullet x=\frac{1}{3}\bullet 5$$
$$x=\frac{5}{3}$$

Validation

$$3x=5$$
$$3\bullet\left(\frac{5}{3}\right)\overset{?}{=}5$$
$$5=5\ \checkmark$$

ERROR: **Incorrect use of Order of Operations**

Incorrect Process

$$4+2x\bullet 3=18$$
$$6\bullet 3x=18$$
$$18x=18$$
$$x=1$$

Resolution

Use the Order of Operations correctly.

Correct Process

$$4+2x\bullet 3=18$$
$$4+6x=18$$
$$6x=14$$
$$x=\frac{14}{6}=\frac{7}{3}$$

Validation

$$4+2x\bullet 3=18$$
$$4+2\bullet\left(\frac{7}{3}\right)\bullet 3\overset{?}{=}18$$
$$4+14\overset{?}{=}18$$
$$18=18\ \checkmark$$

ERROR: **Ignoring a sum of zero**

Incorrect Process

$$3x+13=7$$
$$3x+6$$
$$x=2$$

Resolution

Leave the 0 on the right side of the equation.

Correct Process

$$3x+13=7$$
$$3x+6=0$$
$$3x=-6$$
$$x=-2$$

Validation

$$3x+13=7$$
$$3(-2)+13 \stackrel{?}{=} 7$$
$$-6+13 \stackrel{?}{=} 7$$
$$7=7 \checkmark$$

ERROR: **Adding unlike terms**

Incorrect Process

$$4+2x=18$$
$$6x=18$$
$$x=3$$

Resolution

Use the Addition Property of Equality by adding (–4) to both sides.

Correct Process

$$4+2x=18$$
$$(-4)+4+2x=(-4)+18$$
$$0+2x=14$$
$$2x=14$$
$$x=7$$

Validation

$$4+2x=18$$
$$4+2(7) \stackrel{?}{=} 18$$
$$4+14 \stackrel{?}{=} 18$$
$$18=18 \checkmark$$

Preparation Inventory 2.2

I can...	OR *Here's my question...*
☐ isolate a variable in an equation.	☐
☐ remove parentheses from an equation.	☐
☐ clear fractions in an equation.	☐
☐ solve a linear equation.	☐
☐ solve a literal equation for a given variable.	☐

Critical Thinking Questions 2.2

1. How do you determine which variable to solve for?

2. How do you clear the fractions in an equation?

3. Why eliminate parentheses when solving a linear equation?

4. What are your strategies for isolating the variable term without making a mistake?

5. When isolating the variable, why do you divide both sides of the equation by the coefficient of the variable?

6. How do you make sure that you have isolated the chosen variable correctly?

7. How is isolating a variable different from solving a linear equation?

8. What is the difference between a literal equation and a linear equation?

9. How do the solutions to $2x = 4x - 2x$ and $2x = 4 + 2x$ differ?

Demonstrate Your Understanding 2.2

Solve each literal equation for the indicated variable.

1. Solve for a: $2a + 3 = 7 - 3(2 - 5a)$

2. Solve for x: $5(x - 3) + 4 = -7 + 3x$

3. Solve for x: $-2(x - 5) + \frac{3}{4} = 4x - \frac{5x}{3}$

4. Solve for r: $5 - 3a + 4 = 3 - 2(a + r) - 7$

5. Solve for y: $2(x - 3) = 5 - 3x + y + 5x$

6. Solve for t: $2x - 5t = 3(t - 4) + 7$

Create and Solve the Hardest Problem 2.2

Your Hardest Problem
Problem Solution

What makes a problem with basic equations a hard problem to solve?

Identify, Describe, & Correct the Errors 2.2

Solve the problem correctly in the second column. In the third column, identify and describe the error you found in the worked solution.

Problem 1

Solve for x:

$$x + 5 = 3(4 - 2x)$$

Worked Solution *(Find the error!)*

$$x + 5 = 3(4 - 2x)$$
$$x + 5 = 12 - 2x$$
$$3x = 7$$
$$x = \frac{7}{3}$$

Correct Process

Describe the Error

Problem 2

Solve for x:

$$2x - 5 + x = 6x - 4$$

Worked Solution *(Find the error!)*

$$2x - 5 + x = 6x - 4$$
$$2x + x - 5 = 6x - 4$$
$$3x - 5 = 6x - 4$$
$$3x - 6x = -4 - 5$$
$$-3x = -9$$
$$x = 3$$

Correct Process

Describe the Error

Problem 3

Solve for t:

$$-3^2 + 4s + 5t = 6s - 5s - 4(s + 3 - t)$$

Worked Solution *(Find the error!)*

$$\begin{aligned} -3^2 + 4s + 5t &= 6s - 5s - 4(s + 3 - t) \\ 9 + 4s + 5t &= 6s - 5s - 4s - 12 + 4t \\ 9 + 4s + 5t &= -3s - 12 + 4t \\ 4s + 3s &= -12 + 4t - 9 - 5t \\ 7s &= -12 - 9 + 4t - 5t \\ 7s &= -21 - t \\ s &= -3 - \frac{t}{7} \end{aligned}$$

Correct Process

Describe the Error

Problem 4

Solve for y:

$$\frac{3}{y} - 4 = 3$$

Worked Solution *(Find the error!)*

$$\begin{aligned} \frac{3}{y} - 4 &= 3 \\ 3 - 4y &= 3 \\ -4y &= 0 \\ y &= 0 \end{aligned}$$

Correct Process

Describe the Error

Problem 5

Solve for x:

$$4x + \frac{2}{3}x = 7$$

Worked Solution *(Find the error!)*

$$4x + \frac{2}{3}x = 7$$
$$12x + 2x = 7$$
$$14x = 7$$
$$x = \frac{1}{2}$$

Correct Process

Describe the Error

What Did You Learn About Learning Math? 2.2

1. What prior knowledge did you find useful for learning the material in this section?

2. How did you learn to distinguish equivalence for expressions from equivalence for equations?

3. What are the key questions to ask when solving a linear equation?