Section 2.2

Solving Basic Equations

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$\overline{W}hy?$

You can solve some equations that arise in the real world by isolating a variable. You can use this method to solve the equation $\begin{pmatrix} 1 \end{pmatrix}$

$$400 + \left(1\frac{1}{2}\right)(10)x = 460$$

to determine the number of hours of overtime you need to work to earn \$460 per week, at an hourly rate of \$10, with overtime being time and a half.

What Do You Already Know?

Give two examples of pairs of equivalent equations.

Use the Addition Property of Equality to create an equation that is equivalent to 6+3x=5.

Use the Multiplication Property of Equality to create an equation that is equivalent to -3(x-5) = 6.

Use the Distributive Property and the Substitution Principle to create an equation that is equivalent to 5x + 2(3-x) = 10.

Goals

When you complete this activity, you should be able to:

- 1. Solve a linear equation using the Isolate the Variable Methodology
- 2. Solve a literal equation
- 3. Use the Solving Linear Equations Methodology to find the solution to a linear equation involving fractions and parentheses

Building Mathematica	l Language		2.2
New Terms:	isolate a variable	literal equation	

Key Concepts

Some equations have *no solutions* and some equations have an *infinite number of solutions*. These occur when the variable can be totally eliminated from the equation. The resulting equation will state that two numbers are equal. If the two numbers are different, then the equation is never true and there are no solutions to the original equation. If the two numbers are the same, the equation is always true and there are an infinite number of solutions.

A *literal equation* is an equation with more than one variable. Engineers, scientists, and government policy makers use these equations because they provide information about relationships between real world quantities such as population and the square feet of retail space required.

Limitation: When dealing with literal equations, you must take care to understand the constraints on the variables in the equations. Assuming that literal equations are true for all values of their variables can cause safety issues in energy plants, on planes, and in factories.

Methodology

Isolating a Variable

Isolating a variable in an equation with no fractions or parentheses, and where the variable appears only to the first power, is useful in solving an equation.

Limitation/Caution: If a term has the specified variable in the denominator, then problems can occur (we will see this later when we work with rational equations).

Example 1	Example 2	Your Turn	
Isolate <i>x</i> : $6x - 5 = 7 + 2x$	Isolate <i>x</i> : $6x + 5 - x = 3x + 9$	Isolate <i>a</i> : $2a + 4a = 7a + 2 - 3a$	
Steps	Discussion		
1 Rewrite the equation		write the equation so that all the terms ne side of the equation and all the other uation.	
6x - 2x = 7 + 5	6x - x - 3x = 9 - 5	Your Turn	
2 Simplify the expressions	Simplify the expressions on both	sides of the equation.	
4x = 12	5x - 3x = 4 $2x = 4$	Your Turn	
3 Rewrite the equation	Use the Distributive Property to rewrite the equation so that the chosen variable is a factor.		
Done	Done	Your Turn	
4 Divide by the coefficient	Divide both sides of the equation by the coefficient of the chosen variable.		
4x = 12 $x = 3$	2x = 4 $x = 2$	Your Turn	

Steps	eps Discussion			
5 Validate		Substitute the value of the isolated variable in the equation to verify that the chosen variable has been isolated correctly.		
Ex 1	6x - 5 = 7 + 2x $6(3) - 5 \stackrel{?}{=} 7 + 2(3)$ $18 - 5 \stackrel{?}{=} 7 + 6$ $13 = 13 \checkmark$	6x + 5 - x = 3x + 9 $6(2) + 5 - 2 \stackrel{?}{=} 3(2) + 9$ $12 + 5 - 2 \stackrel{?}{=} 6 + 9$ $17 - 2 \stackrel{?}{=} 15$ $15 = 15 \checkmark$		
Your Turn				

Model 1: Isolating a Variable in a Literal Equation

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Isolate y^2 : $3y^2 - 4ax = 7ax - 5 + ay^2$

Step 1	Rewrite the equation	$3y^2 - ay^2 = 7ax - 5 + 4ax$
Step 2	Simplify	$3y^2 - ay^2 = 11ax - 5$
Step 3	Rewrite with chosen variable as a factor	$y^2(3-a) = 11ax - 5$
Step 4	Divide by the coefficient	$\frac{y^2(3-a)}{(3-a)} = \frac{11ax-5}{(3-a)}$
		$y^2 = \frac{11ax - 5}{3 - a}$

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Step 5 Validate $3y^{2} - 4ax = 7ax - 5 + ay^{2}$ $3\left(\frac{11ax - 5}{3 - a}\right) - 4ax = 7ax - 5 + a\left(\frac{11ax - 5}{3 - a}\right)$ $\frac{33ax - 15}{3 - a} - 4ax\left(\frac{3 - a}{3 - a}\right) = 7ax\left(\frac{3 - a}{3 - a}\right) - 5\left(\frac{3 - a}{3 - a}\right) + \frac{11a^{2}x - 5a}{3 - a}$ $\frac{33ax - 15 - 12ax + 4a^{2}x}{3 - a} = \frac{21ax - 7a^{2}x - 15 + 5a + 11a^{2}x - 5a}{3 - a}$ $\frac{4a^{2}x + 21ax - 15}{3 - a} = \frac{4a^{2}x + 21ax - 15}{3 - a}$

Methodology

Clearing Fractions

Equations without denominators (fractions) are easier to work with.

Limitation: Some denominators with variables can cause problems.

	Example 1	Your Turn
Clear the fractions:	$\frac{x}{3} + \frac{2x}{5} = 4$	$\frac{3a}{4} + \frac{a^2}{5} = 3$

Steps	Discussion
1 Determine LCD	Determine the LCD of all the denominators of the equation.
$\frac{x}{3} + \frac{2x}{5} = 4$ The LCD is 3 • 5	Your Turn
2 Multiply and simplify	Multiply both sides of the equation by the LCD of the denominators in the equation and simplify.
$15\left(\frac{x}{3} + \frac{2x}{5}\right) = 15(4)$ $15\left(\frac{x}{3}\right) + 15\left(\frac{2x}{5}\right) = 60$ $\frac{15x}{3} + \frac{30x}{5} = 60$ $5x + 6x = 60$	Your Turn

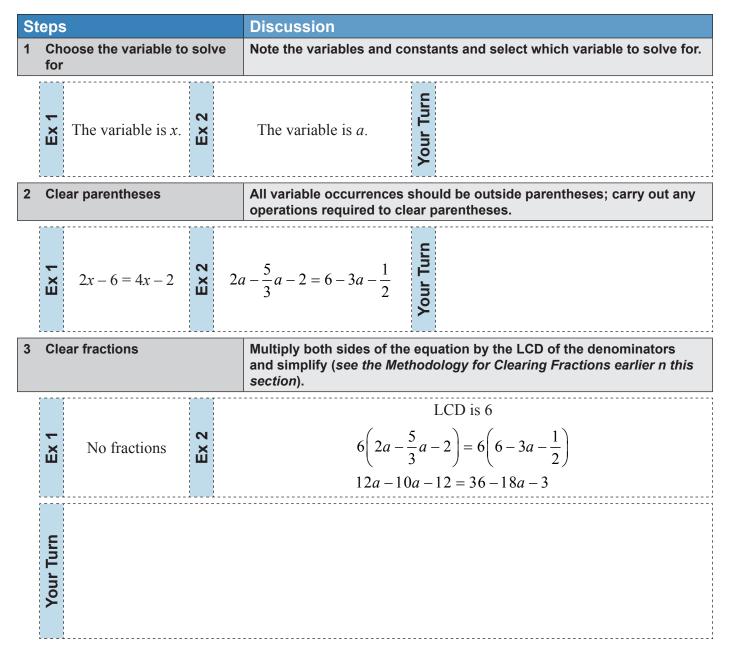
Methodology		2.2
	Solving a Linear Equation	

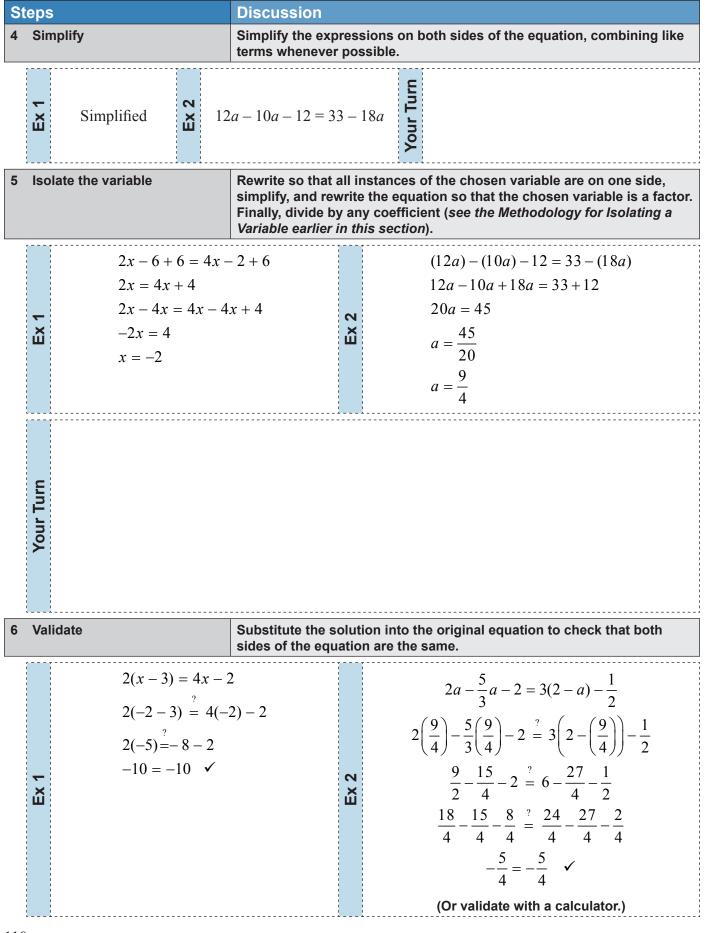
A linear equation in one variable defines possible values for that variable, and can be used to find these values. Linear equations that state a relationship between two or more variables can be solved to determine the relationship one variable has to the other variable(s).

Limitation/Caution: Although some relationships defined by data appear linear for a range of values of the variable(s), assuming that a relationship is linear can lead to errors.

Solve the linear equation for the designated variable:

Example 1	Example 2	Your Turn
for <i>x</i> : $2(x-3) = 4x - 2$	for <i>a</i> : $2a - \frac{5}{3}a - 2 = 3(2 - a) - \frac{1}{2}$	for <i>t</i> : $6 - 3(t - 5) = 2t + 11$





Steps	Discussion	
Your Turn		
Model 2: Solving a Literal	Equation for a Given Variable 2.2	2

Solve the following literal equation for *r*:

$$\frac{7}{3} - 2rw = \pi r$$

Step 1	Choose the variable to solve for	The variables are <i>r</i> and <i>w</i> . The symbol π is a constant. We are to solve for <i>r</i> .
Step 2	Clear parentheses	There are no parentheses.
Step 3	Clear fractions	$\frac{7}{3} - 2rw = \pi r$ LCD is 3 $7 - 6rw = 3\pi r$
Step 4	Simplify	There is nothing to simplify.
Step 5	Isolate the variable	$7 - (6rw) = (3\pi r)$ $7 = 3\pi r + 6rw$ $7 = (3\pi + 6w)r$ $\frac{7}{(3\pi + 6w)} = r$ $\frac{7}{3(\pi + 2w)} = r \text{or} r = \frac{7}{3(\pi + 2w)}$

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Step 6 Validate

$$\frac{7}{3} - 2rw = \pi r$$

$$\frac{7}{3} - 2\left(\frac{7}{3(\pi + 2w)}\right) \cdot w = \pi \left(\frac{7}{3(\pi + 2w)}\right)$$

$$\frac{7}{3} - \frac{14w}{3(\pi + 2w)} = \frac{7\pi}{3(\pi + 2w)}$$

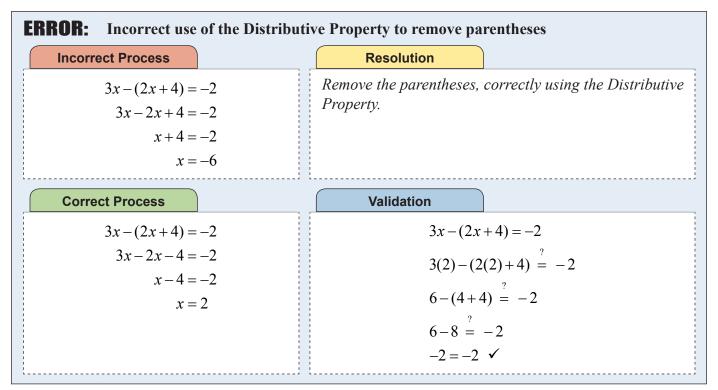
$$\frac{7(\pi + 2w) - 14w}{3(\pi + 2w)} = \frac{7\pi}{3(\pi + 2w)}$$

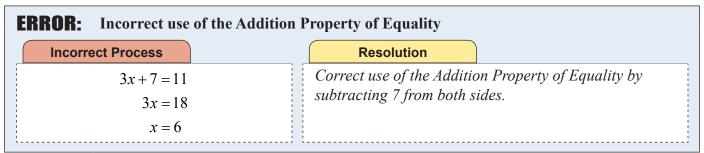
$$\frac{7\pi + 14w - 14w}{3(\pi + 2w)} = \frac{7\pi}{3(\pi + 2w)}$$

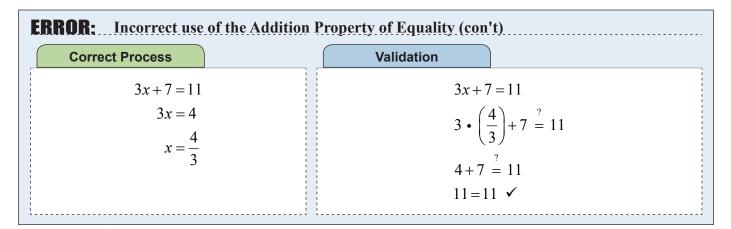
$$\frac{7\pi}{3(\pi + 2w)} = \frac{7\pi}{3(\pi + 2w)} \checkmark$$

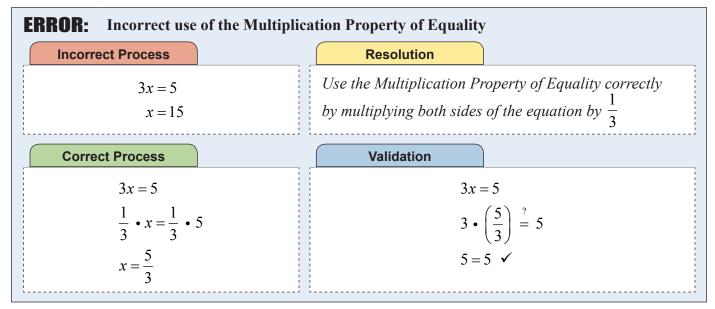
Addressing Common Errors

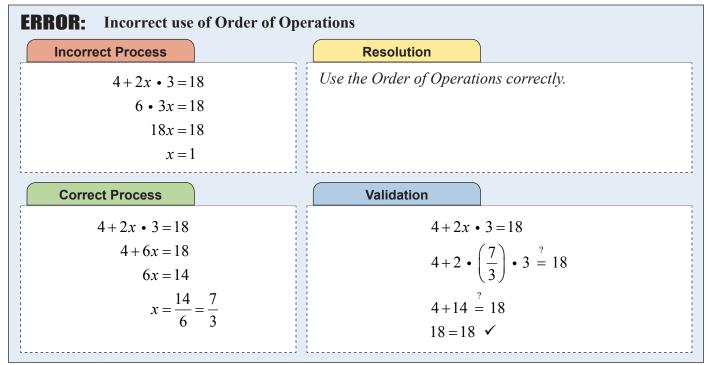
The following table shows some of the most common errors that learners tend to make with the kinds of problems covered in this section.

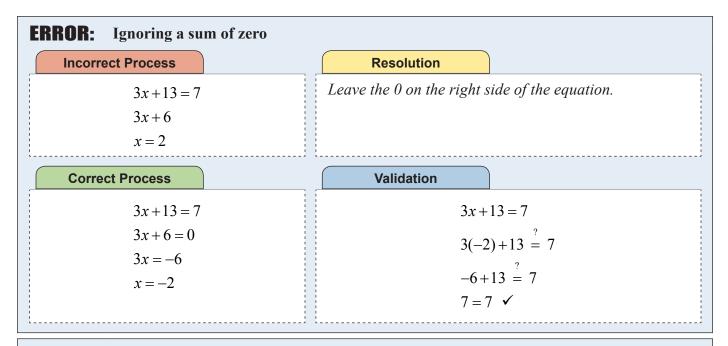












ERROR: Adding unlike terms

Incorrect Process	Resolution	
4 + 2x = 18 $6x = 18$	Use the Addition Property of Equality by adding (–4) to both sides.	
x = 3		
Correct Process	Validation	
4 + 2x = 18	4 + 2x = 18	
(-4) + 4 + 2x = (-4) + 18	$4+2(7) \stackrel{?}{=} 18$	
0 + 2x = 14	?	
2x = 14	4 + 14 = 18	
<i>x</i> = 7	18 = 18 ✓	

Preparation Inventory

I can	OR	Here's my question
isolate a variable in an equation.		
remove parentheses from an equation.		
clear fractions in an equation.		
solve a linear equation.		
solve a literal equation for a given variable.		

Critical Thinking Questions

1. How do you determine which variable to solve for?

2. How do you clear the fractions in an equation?

3. Why eliminate parentheses when solving a linear equation?

4. What are your strategies for isolating the variable term without making a mistake?

5. When isolating the variable, why do you divide both sides of the equation by the coefficient of the variable?

6. How do you make sure that you have isolated the chosen variable correctly?

7. How is isolating a variable different from solving a linear equation?

8. What is the difference between a literal equation and a linear equation?

9. How do the solutions to 2x = 4x - 2x and 2x = 4 + 2x differ?

Demonstrate Your Understanding

Solve each literal equation for the indicated variable.

1. Solve for a: 2a + 3 = 7 - 3(2 - 5a)

2. Solve for *x*: 5(x-3) + 4 = -7 + 3x

3. Solve for x:
$$-2(x-5) + \frac{3}{4} = 4x - \frac{5x}{3}$$



4. Solve for *r*: 5 - 3a + 4 = 3 - 2(a + r) - 7

5. Solve for *y*: 2(x-3) = 5 - 3x + y + 5x

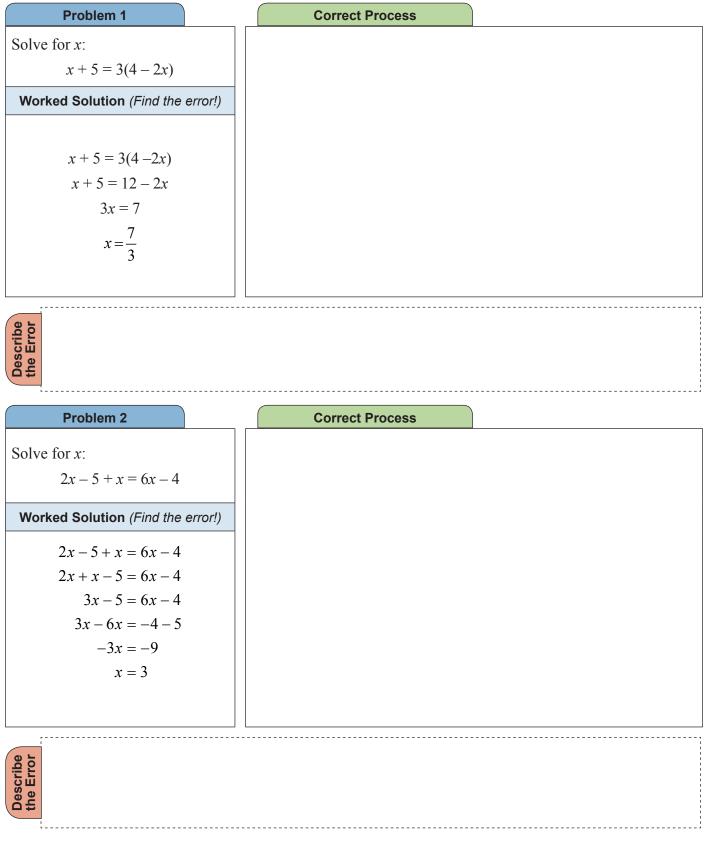
6. Solve for t: 2x - 5t = 3(t - 4) + 7

Create and Solve the Hardest Problem	2.2
Your Hardest Problem	
Problem Solution	

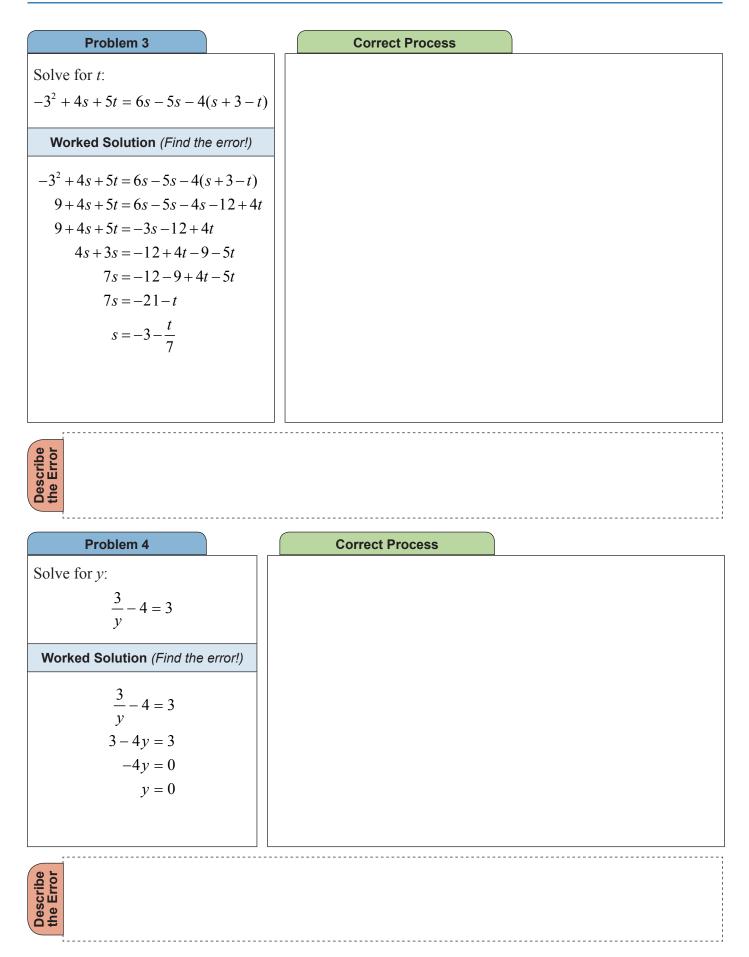
What makes a problem with basic equations a hard problem to solve?

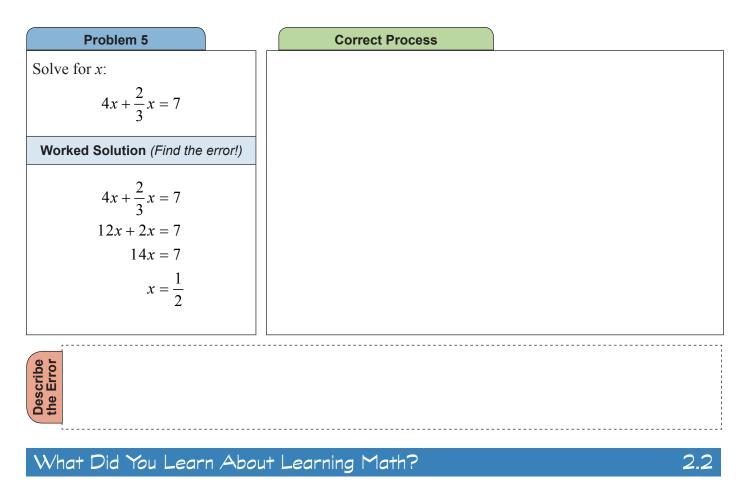
Identify, Describe, & Correct the Errors

Solve the problem correctly in the second column. In the third column, identify and describe the error you found in the worked solution.



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1. What prior knowledge did you find useful for learning the material in this section?

2. How did you learn to distinguish equivalence for expressions from equivalence for equations?

3. What are the key questions to ask when solving a linear equation?