Precision, Accuracy, and Clarity

The following table offers examples of English and mathematical statements that demonstrate differing degrees of precision, accuracy, and clarity. It is important to note that precision and accuracy are often treated the same in English; we tend to use the words interchangeably. They have different meanings in mathematics, however. An inaccurate answer in mathematics is an error. A mathematical solution may be very precise and still be inaccurate. For example: $16 \div 3.8 \approx 4.220528$. The correct answer, to the same number of decimal places (one way to measure the degree of precision) is: 4.210526.

	English		Mathematics	
	Less	***More***	***Less***	***More***
Precision	Sarah and her best friend went to a movie.	Sarah and Jane went to see the latest blockbuster.	$\frac{1}{3} \approx 0.3$	$\frac{1}{3} \approx 0.3333$
Accuracy	I didn't hear my alarm go off.	I heard my alarm go off, but hit the snooze button and fell back asleep.	$\pi \approx 3.15$	$\pi \approx 3.14$
Clarity	He feels bad, coughing and stuff like that, you know—sick.	He does not feel well, has a cough, and other symptoms of illness.	$\left[\left(9 \times \sqrt{4}\right) + \left(\frac{3}{2}\right)\right] - \left(1^2\right)$	18.5

In both English and mathematics, the more precise a statement, the more likely it is that someone else can interpret it accurately. One of the most common ways that inaccuracies are introduced in mathematics is through a lack of neatness. The more neatly you write as you perform calculations or when you present your work, the less likely it is that you will misread your writing (mistaking "2" for "z" for example) or have your work misread by others.

Clarity is a kind of neatness of thinking or process. The clearer your understanding, the more accurate your work will be. In the process of communication, clarity means that what you say or write is easily understood by others. Consider how frustrating it is to follow unclear directions when you're trying to get somewhere. The clearer the directions, the better your chances of arriving at the accurate destination. (Precision helps here too! Being told to "turn left after a few blocks" isn't very helpful.) In all mathematics courses, as well as other courses which use mathematics, you will be asked to "show your work." This really means *to make clear the reasoning you used to arrive at your answer.* Your reasoning and its clarity are as important as the accuracy of your answer.

In this course, we value precision and clarity, as they help to eliminate ambiguity and produce accuracy in all our work.

When we are confronted with word problems that must be solved mathematically, we must translate the meaning of the problem into mathematical notation. This does **not** mean assigning a mathematical symbol or number to individual words (*transliteration*); doing this is guaranteed to lead to wrong answers. The act of translating requires that you first *understand* the meaning of the problem in English. Then you *interpret* that meaning, selecting and ordering mathematical symbols and numbers that represent the same meaning. Only then can you work to solve the problem.

In translating word problems into precise mathematical language, we want to ***eliminate ambiguity*** completely. This means that we must write mathematical statements that precisely, accurately, and clearly reflect the meaning of the English language description of the problem situation. Inaccuracies can be introduced into mathematical statements by not adhering to conventions agreed upon by mathematicians. For example, 20% of 75 is 15, because the convention **in this particular situation** is "*of* means multiplication." However, the phrase "5 of 8" may mean several different things. Just by adding one word here or there, you can drastically change the meaning of the statement. Consider the following examples, where the meaning for the word ***of*** differs:

Statement	Context	Mathematical meaning
5 chances **of 8** chances	What fraction of the time will you win a lottery if you hold 5 of 8 winning tickets?	5/8 of the time you will win the lottery
5 teams **of 8** players	What is the total number of players?	$5 \times 8 = 40$ players
5 percent **of 8** dollars	How much is the discount on purchasing an $8 item?	$5\% \times \$8 = 0.05 \times 8 = \$\,0.40$ or 40¢
5 slices out **of 8** slices	How much of a pizza pie has been eaten?	5/8 of the pizza pie
choose **5** elements out **of 8**	How many different 5 member committees can be chosen from 8 possible members?	${}_8C_5$ possible committees

That the meaning of words (such as *of*) varies by context is why *transliteration* (substituting mathematical symbols for words) causes errors.

When you translate statements such as those in the first column of the table above into mathematical language, you must use mathematical symbols that communicate the ***meaning*** of the English statement, not the individual words. This meaning is based on your understanding and interpretation of the statement. When there is ambiguity in an English language presentation of a situation, you can do one of two things. You can choose to interpret the statement based on inference (a tentative conclusion based on clues) or you can ask for more information.

Caution: Mathematical conventions do not exhaustively cover all English language usage. You cannot be sure that the person who wrote a problem statement in English used or even knew any of the conventions established by mathematicians. This means that when ambiguity exists, you may not correctly understand and interpret the context or meaning intended by the writer.

The methodologies in this book have been created by experts as a way to help others learn to achieve clarity in mathematical reasoning that leads to accuracy in worked-out solutions. Your use of these methodologies can help you increase the clarity of your reasoning and work, therefore helping you reduce misinterpretations and inaccuracies.

Caution: A methodology is simply a model of what experts feel are the critical steps in mathematical reasoning that are needed to solve a given type of problem.