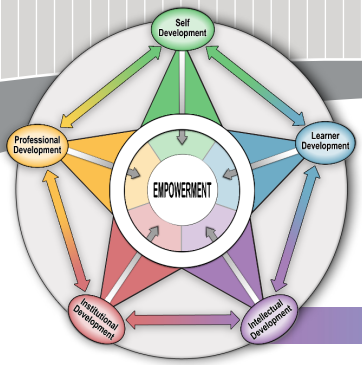
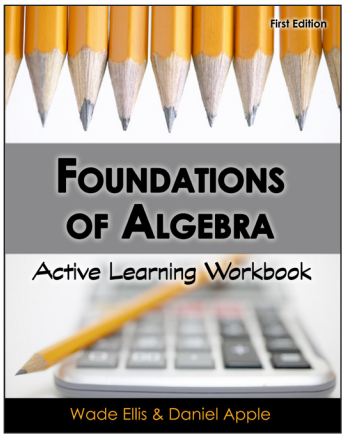


Course Design Information for
FOUNDATIONS OF ALGEBRA



1. **Long-term Behaviors** *What the student should learn to practice in life, personally & professionally*
2. **Broad Learning Goals** *Key learning objectives*
3. **Course Intentions** *Intended results of the course*
4. **Learning Outcomes** *What the student should have learned at the end of the course; measurable*
5. **Knowledge Table** *Types of knowledge the student should master to achieve learning outcomes*
6. **Methodologies** *Explicit models of key practices*
7. **Themes** *Support the development of long-term behaviors; connect multiple activities*
8. **Learning Skills** *Transferable skills that improve the student's ability to learn across contexts*
9. **Activity Table** *Learning activities developed for achievement of learning outcomes*

1. Long-Term Behaviors promoted by *Foundations of Algebra*

1. **Algebraist**: Has a working expertise of algebraic skills that allows easy manipulation of any expression, equation or function by analyzing, expanding, simplifying or solving in any disciplinary context through the use of their conceptual understanding of algebraic principles and laws
2. **Mathematical Thinker**: Sees mathematical structures, relationships, and commonalities that make future learning quicker, stronger, and contextually relevant by learning to generalize mathematical knowledge and thus leveraging prior knowledge in new learning
3. **Life-long Learner**: Has developed and uses strong mathematical learning skills that align with their long-term learning plan and leverages daily mathematical situations as opportunities for learning and mathematical development
4. **Mathematical Modeler**: Is purposeful in thought and takes in ideas and models from a variety of people and sources, identifies key variables and relationships, makes solid connections and synthesizes them into a simple, coherent and well developed framework.
5. **Problem Solver**: Locates and identifies key problems in life situations, clearly defines key issues and working assumptions, systematically breaks down the problem into manageable sub problems and integrates known workable algebraic solutions of sub problems into a validated and documented solution that has been generalized across additional opportunities.
6. **Reflective Practitioner**: Values and practices both self-assessment and reflection to help personally and professionally to improve performance and the quality of life based upon both personal and professional values, and takes these skills and helps other to improve their performance in mathematics through quality peer-assessment, mentoring, and systematic continuous quality improvement.
7. **Mathematical Communicator**: Can produce strong mathematical language structure, reasoning, precision, and appropriateness so others understand exactly the quality of validated logical development produced.

2. Broad Learning Goals of *Foundations of Algebra*

- Advance problem solving process
- Connect math to problem solving
- Explore and improve performance in elevating mathematical learning to level 4 on Bloom's
- Build a reflective mindset with practical tools
- Building and applying algebraic models
- Increase mathematical reasoning
- Strengthen algebraic tool set across contexts

3. Course Intentions of *Foundations of Algebra*

- To produce a new way of learning mathematics to increase its value and relevance to students in their discipline and in life
- Increase their efficacy in learning mathematics
- Build a problem solving mindset
- Demonstrate that algebra can be learned quickly and effectively by anyone
- Reduce the number of developmental courses in community colleges that a student has to take to just one course in order to take transfer credit math courses

4. Learning Outcomes for *Foundations of Algebra*

Competencies *The collection of knowledge, skills, and attitudes needed to perform a specific task effectively and efficiently at a defined level of performance*

- Produce an effective reading log of a mathematical text
- Produced a documented solution to an algebraic challenge
- Can analyze a mathematical model to bring meaning and do what-if scenarios based upon changing variables.
- Can read, interpret and produce a wide-range of functional and analytical graphs
- Can use a reflective journal for consistent assessment of one's own learning processes so that learning performance can continue to improve
- Can simplify, expand, reduce or evaluate any expression
- Can rearrange, graph or solve any standard algebraic equation
- Can graph and analyze any common algebraic function to describe behavior over its domain and range

Movement *Documented growth in a transferable process or learning skill*

- Problem Solving: Advancement in identifying, defining, clarifying issues and assumptions, partitioning, modeling, integrating, testing, generalizing, and documenting problem solving process
- Learning Math: Identifying the general structure, clarifying the key principles, obtaining and processing examples or models of the principles, knowing the underlying fundamental assumptions and relationships, and testing understanding by contextualizing and applying to familiar and unfamiliar situations to build transfer skills
- Modeling: defining a system, its key components, the significant variables, inputs and outputs of the system, diagramming the system, and describing the relationships both in visual and symbolic forms.
- Self-development: through ongoing reflections and self-assessment, identify key areas for growth, identify mentors, structure self-improvement projects, assess progress (strengths, accomplishments, and areas for improvement), and celebrate growth with those who have helped you along the way

4. Learning Outcomes for *Foundations of Algebra (con't)*

Experience	*Interactions, emotions, responsibilities, and shared memories that clarify one's position in relation to oneself, a community, or discipline*

Students will be given challenging learning situations that require both collaboration and cooperation to meet a set of performance criteria in a public arena, where the performance is interdependent on others' performance. You then will be given an increasingly more difficult set of algebraic problems to solve individually as well as in teams. These problems will be assessed and improved upon through peer-assessment. The results of your efforts will always be public with the opportunity for assessment that can be integrated into future plans for growth. Through these algebraic challenges, you will feel many ups and downs that are fairly significant and gives you opportunity to grow your emotional maturity and affective skill set with regards to mathematical efficacy. The learning processes and the environment created will expect you to be professional consistent, coming to activities well-prepared, seeking to be at peek performance, and reflecting on practice to help yourself and others to improve daily performance in mathematical learning. The enriched learning environment will constantly challenge you to seek higher levels of learning and developing action plans for improvement and growth.

Achievement	*Significant work products or performances that transcend normal class requirements and are externally valued or affirmed by an outside expert or client*

Create a mathematical solution for a non-profit organization by solving a complex problem through building a mathematical model of their situation

Integrated Performance	*The synthesis of prior knowledge, skills, processes, and attitudes with current learning needs to address a difficult challenge within a strict time frame and set of performance expectations*

Independently tackles a new area of knowledge with purpose and direction by

- identifying the learning outcomes
- the basic structure and language for the disciplinary knowledge
- are proficient readers to gain independent meaning through strong inquiry questions
- know how to use models and examples to contextualize and generalize their knowledge
- integrates new and past knowledge through applying this knowledge in difficult problem solving situations
- secures knowledge by validating learning by knowing that you know

5. Knowledge Table for *Foundations of Algebra*

Concept: *an idea that connects a set of relationships; a generalized idea about something*
Process: *a sequence of steps, events, or activities that results in a change or produces something over time*
Tool: *any device, implement, instrument, or utensil that serves as a resource to accomplish a task*
Contexts: *the whole situation, background, or conditions relevant to the process*
Way of Being: *the set of behaviors, actions, & language associated with a particular discipline or knowledge area; a culture*

Concepts	Processes	Tools	Context	Way of Being
Variable Expression Mathematical language Equation Function Equivalence Number system Inequalities Inverse Functions Polynomial arithmetic Methodology Substitution Slope-rate of change Co-variation Convention	Problem solving methodology Interpreting a Math model Validation Learning Process methodology Self-assessment/ reflection Reading process Interpreting Word Problems Solving an equation Simplifying Analyzing a function Expanding evaluating Factoring	Graphing Coordinate System Family of Function Cards T-Table Properties: exponents, logs, equations, inequalities, real numbers, radicals, polynomials, complex numbers Laws: commutative, associate and distributive, identities, Mathematical software/ calculators Concept Map	Logarithmic Equations Rational Equations Exponential Equations Polynomial Equations Radical Equations Systems of equations Translating - numerical, graphical and symbolic representation Absolute value Equation Quadratic Equation	Persistence Risk taking Validation Problem Solver Efficacy

6. Core Methodologies

- Problem Solving
- Learning Process
- Assessment
- Solving an Equation
- Reading
- Simplifying
- Expanding
- Graphing
- Analyzing a Function
- Evaluating an Equation
- Interpreting Word Problems
- Validating
- Factoring

7. Themes

- Quantitative Problem Solving
- Learning to Learn Math: Reading and Learning
- Mathematical Reasoning and Thinking
- Mathematical Modeling
- Problem Solving
- Self-growth, Self-assessment, and CQI

8. Learning Skills for *Foundations of Algebra*

1. **Identifying similarities**: recognizing common attributes of parts
2. **Identifying differences**: recognizing/distinguishing attributes of parts
3. **Recognizing patterns**: perceiving consistent repetitive occurrences
4. **Identifying assumptions**: examining preconceptions/biases
5. **Inquiring**: asking key questions
6. **Exploring context**: seeing the relationship of parts to the environment
7. **Bounding**: recognizing the limits of the application of knowledge
8. **Transferring**: using ideas in a new context
9. **Simplifying**: representing only primary features
10. **Complying**: comparing results with accepted standards
11. **Defining the problem**: articulating a problem and need for solution
12. **Generalizing**: transferring knowledge to multiple contexts
13. **Abstracting**: describing the essence of an idea, belief, or value
14. **Validating**: ensuring the quality of the mathematical reasoning
15. **Visualizing**: translating the mathematical idea via a picture
16. **Documenting**: precisely in writing trace the mathematical reasoning
17. **Persisting**: continuing despite difficulties
18. **Using prior knowledge**: integrating unprompted knowledge

9. Activity Table for *Foundations of Algebra*

No.	ACTIVITY NAME	KNOWLEDGE TBL ITM	THEME	ACTIVITY TYPE	LEARNING SKILLS
1.1	Expanding the Number System	Laws: commutative, associative, and distributive, identities; Number system;	Learning to Learn Math: Reading and Learning	Guided Discovery Learning	Using Prior Knowledge Identifying differences Transferring
1.2	Working with Numbers Represented by Radicals	Properties: Radicals; Validation	Learning to Learn Math: Reading and Learning	Interactive Lecture	Transferring Bounding Recognizing patterns
1.3	Working with Complex Numbers	Properties: Complex Numbers; Number system; Simplifying	Learning to Learn Math: Reading and Learning	Interactive Lecture	Transferring Identifying similarities Abstracting
1.4	Interpreting Word Problems	Interpreting Word Problems; Variable	Quantitative Problem Solving	Problem Solving	Defining the problem Identifying assumptions Documenting
1.5	Simplifying Algebraic Expressions	Simplifying; Mathematical Language; Convention	Mathematical Reasoning and Thinking	Group Discussion/ Dialog	Identifying similarities Simplifying Complying
1.6	Exponents and Expanding Expressions	Properties: Exponents, Logs; Expanding; Expression; Equivalence	Mathematical Reasoning and Thinking	Interactive Lecture	Recognizing patterns Using prior knowledge Bounding
1.7	Evaluating Formulas	Evaluating	Mathematical Modeling	Students Teaching	Exploring context Simplifying Complying
2.1	Equations: Equivalence and Substitution	Equations; Equivalence; Substitution; Properties: Equations, Exponents	Learning to Learn Math: Reading and Learning	Interactive lecture; Collaborative Learning	Visualizing Identifying similarities Identifying differences
2.2	Solving Basic Equations	Equivalence; Solving an Equation; Properties: Equations	Learning to Learn Math: Reading and Learning	Self-study; Assessment/Peer Assessment	Inquiring Visualizing Documenting
2.3	Solving Systems of Equations	Solving Systems of Equations; Substitution; Variable; Translating: Symbolic, numerical, graphical	Mathematical Modeling OR Mathematical Reasoning and Thinking	Interactive Lecture; Assessment/Peer Assessment	Exploring context Transferring Visualizing
2.4	Validation	Validation; Problem Solving; Efficacy	Mathematical Reasoning and Thinking	Technology; Writing	Inquiring Complying Documenting
3.1	Solving and Graphing a Linear Inequality of a Single Variable	Inequalities; Properties: Inequalities; Graphing	Learning to Learn Math: Reading and Learning	Reading; Group Discussion/Dialog	Visualizing Transferring Complying

No.	ACTIVITY NAME	KNOWLEDGE TBL ITM	THEME	ACTIVITY TYPE	LEARNING SKILLS
3.2	Solving and Graphing Compound Inequalities of a Single Variable	Inequalities; Graphing	Mathematical Reasoning and Thinking	Interactive Lecture	Visualizing Generalizing Identifying similarities
3.3	Graphing Equations Using a Table	Graphing; T-Table; Coordinate System	Learning to Learn Math: Reading and Learning	Guided Discovery Learning	Recognizing patterns Exploring context Complying
3.4	Graphing Equations in Two Variable Using Slope	Slope-rate of change; Graphing	Mathematical Modeling	Guided Discovery Learning	Abstracting Visualizing Generalizing
3.5	Graphing Other Equations in Two Variables	Graphing; Variable; Mathematical software/calculator; Translating: symbolic, numerical, graphical	Mathematical Reasoning and Thinking	Technology; Demonstration	Identifying assumptions Inquiring Bounding
3.6	Solving Inequalities in Two Variables	Inequalities	Mathematical Reasoning and Thinking	Cooperative Learning	Identifying similarities Identifying differences Bounding
4.1	Dividing Polynomials	Polynomial arithmetic	Learning to Learn Math: Reading and Learning	Interactive Lecture	Recognizing patterns Using prior knowledge Documenting
4.2	Factoring Trinomials Polynomials	Factoring; Polynomials; Validation	Mathematical Reasoning and Thinking	Student Communication; Presentations	Defining the problem Generalizing Recognizing patterns
4.3	Factoring Other Polynomials	Factoring; Polynomials	Mathematical Reasoning and Thinking	Self-Study	Inquiring Recognizing patterns Simplifying
4.4	Simplifying Rational Expressions Are 21 and 22 reversed?	Expression; Simplifying;	Mathematical Reasoning and Thinking	Cooperative Learning	Simplifying Complying Bounding
4.5	Solving Quadratic and Other Polynomial Equations	Quadratic Equations; Polynomial Equations; Mathematical software/calculators	Mathematical Reasoning and Thinking	Interactive Lecture; Technology	Simplifying Recognizing patterns Documenting
5.1	Basics of Functions	Function	Learning to Learn Math: Reading and Learning	Interactive Lecture	Recognizing patterns Visualizing Exploring context
5.2	Families of Functions	Families of Function Cards; T-Table; Graphing; Validation, Slope-rate of change; Interpreting a Math Model; Co-variation	Mathematical Modeling	Student Communication/ Presentations	Visualizing Abstracting Generalizing
5.3	Analyzing a Function	Analyzing a Function; Graphing; Mathematical software/calculator	Mathematical Reasoning and Thinking	Technology; Assessment/Peer Assessment	Identifying similarities Identifying assumptions Persisting Bounding

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No.	ACTIVITY NAME	KNOWLEDGE TBL ITM	THEME	ACTIVITY TYPE	LEARNING SKILLS
5.4	Inverse Relations and Functions	Inverse Functions; T-Table; Graphing	Mathematical Reasoning and Thinking	Interactive Lecture	Visualizing Simplifying Recognizing patterns
5.5	Manipulating Functions and the Basics of Comparing Functions	Analyzing a Function	Mathematical Reasoning and Thinking	Collaborative Learning; Journaling	Identifying differences Visualizing Abstracting
6.1	Solving Absolute Value and Rational Equations	Absolute Value Equation; Rational Equation; Solving an Equation	Mathematical Reasoning and Thinking	Collaborative Learning; Role Playing	Defining the problem Transferring Validating
6.2	Transforming Equations	Substitution; Properties: Equations; Equivalence; Inverse Functions	Mathematical Reasoning and Thinking	Interactive Lecture	Visualizing Bounding Inquiring
6.3	Solving an Equation	Solving an Equation; Rational, Exponential, Logarithmic, Radical, Power, Absolute Value Equations	Mathematical Reasoning and Thinking; Mathematical Modeling	Reading; Guided Discovery Learning	Identifying similarities Identifying differences Exploring context
6.4	Additional Tools for Solving Equations		Mathematical Modeling	Problem-based Learning; Technology	Defining the problem Exploring context Validating

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